On the reasons of scatter in data for design formula evolution

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Abstract

The variety of design formulas for the determination of wave action (run-up, overtopping) at coastal structures indicates uncertainties in the interpretation of results from hydraulic and numerical model tests. Two sources of data scatter are discussed in the following:

- **varying statistics of wave heights in the irregular wave trains used in the physical models**
- **the use of the reference wave parameters at the toe of sloped sea dikes, recommended in the more recent literature without considering energy flux (shoaling) and possibly refraction effects.**

To quantify the influences of wave irregularity, overtopping rates from regular wave tests were used in combination with the probability calculation method to calculate irregular wave results at vertical walls. The range of the influence of the reference wave height parameter in case of sloped structures is demonstrated by varying the reference water depth and applying the currently recommended design formula from EurOtop; furthermore by measurements in regular waves. The influence on wave run-up in oblique waves is demonstrated on the basis of measurements in irregular waves.

Keywords: Data scatter, Design formulas, Overtopping, Run-up, Model tests

I. Examples of data scatter in overtopping measurements

Design rules and recommendations are mostly developed from investigations with irregular waves in hydraulic models. The results are usually presented as dimensionless mean overtopping rates as function of the dimensionless freeboard. Definitions for the dimensionless parameters are dependent on the type of the structure and the hydraulic conditions. The design formulas are determined by fitting of suitable functions to the data. Often an exponential relationship between dimensionless mean overtopping rate and dimensionless freeboard is hypothesised. This function is a straight line when the vertical axis is plotted with logarithmic scale. In Figure 1 results from overtopping measurements at vertical walls and various design formulas derived from such data are plotted. For sloped sea dikes in breaking waves an example from EurOtop is shown in Figure 2. Both examples show that there is a considerable scatter in the data. This can be seen as an explanation for the variety of design formulas. However, this should not be accepted without doubtless attribution to physical reasons.

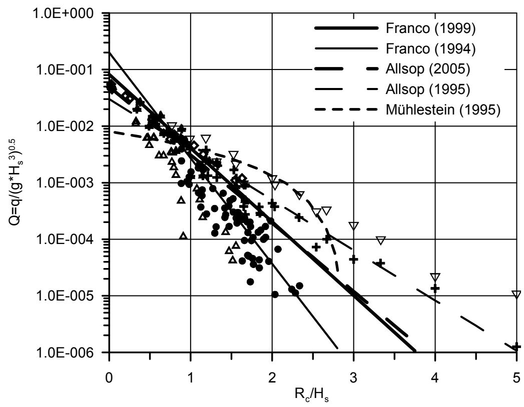

Figure 1. Some data sets and design formulas on wave overtopping at vertical walls

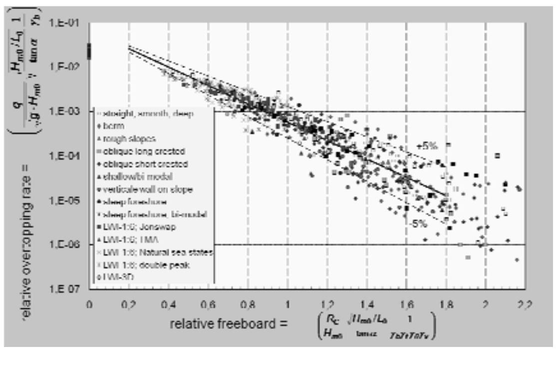

Figure 2. Data and design formula for sloped sea dikes (from EurOtop, 2002)

II. Scatter due to varying statistics of wave heights in the irregular wave trains

A. Generation and characteristics of wave trains in models

Wave trains to be used in models are typically generated by inverse Fourier-transformation from a theoretical spectrum (example Fig. 3).

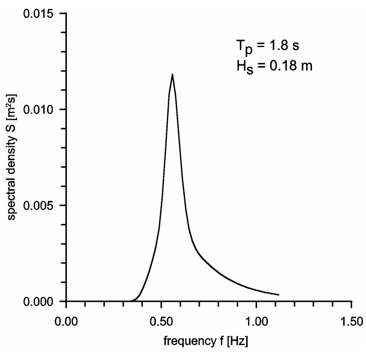

Figure 3. JONSWAP-spectrum ($H_s = H_{m0}$ = 0.18 m, T_p = 1.8 sec)

With selecting the spectrum, the significant spectral wave parameters H_{m0} and T_p are fixed. At first the modeller chooses the length of the time series, which finally decides on the number of individual zero-crossing waves. This length is controlled by the frequency spacing of the Fourier components in the discrete spectrum of amplitudes. In the following example the frequency spacing was selected to become a time-series of about 58 s with about 35 zero-crossing waves. The characteristics of zero-crossing wave heights and periods results from the phase angels attributed to the components. These are usually taken as random. In Fig. 4 the discrete spectrum of amplitudes, a set of random phase angles and the relating wave time-series for this set of phase angles are plotted.

Different "seeds" of random phase angles result in different time-series. The random phases control or determine finally the zero-crossing characteristics of the individual waves (examples Fig. 5).

Analysing such time-series in detail we find variations in the distribution of the high waves and especially a variation of the maximum wave in the various wave-trains from same spectral density distributions. These variations in the distributions affect the model results, particularly in investigations of non-linear processes as e.g. wave overtopping (however, this occurs similarly in natural wave trains). The variations of the wave height distributions and therewith model test results can be very much dependent on the number of waves in the model wave train. A general recommendation of 1000 waves does not meet the requirements in individual cases. Time-series with fewer waves might be sufficient or be indispensable, because of reflections in models or computing time.

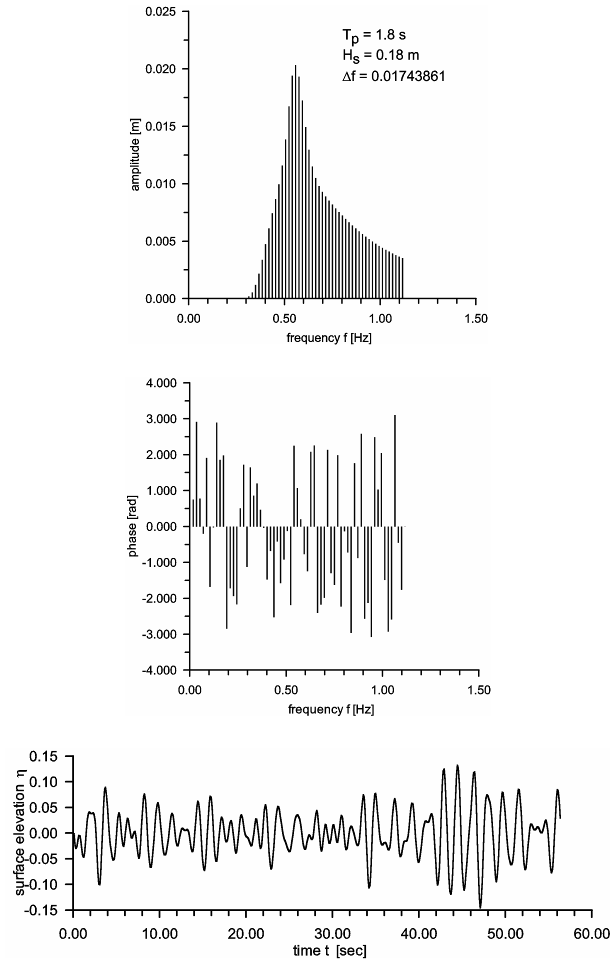

Figure 4. Discrete spectrum of amplitudes, phase angles (random), and resulting wave time-series

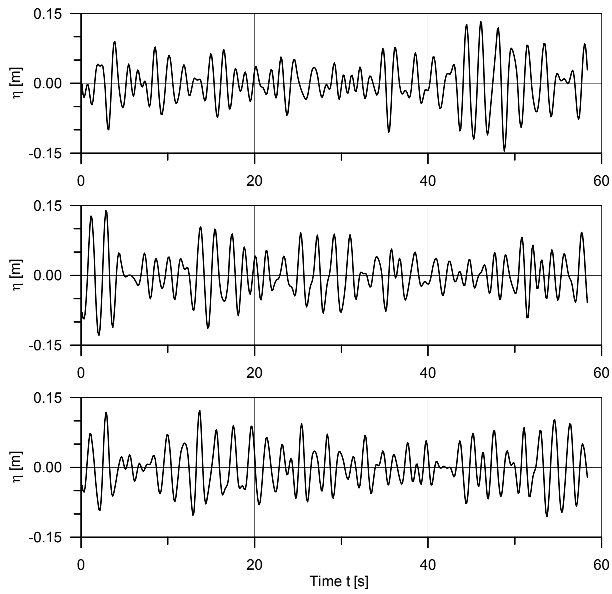

Figure 5. Examples of wave time-series from various random seeds of phases and same spectrum

B. Scatter in mean overtopping rates at vertical walls

The importance of the subject is demonstrated by mean overtopping rates at vertical walls as a function of the freeboard.

The overtopping rates were calculated on the basis of the "probability calculation method" [1]. In this method each wave (zero-crossing definition) in a wave train is treated as an independent regular wave with overtopping volumes according to regular wave tests.

Extensive physical model tests with regular waves at vertical walls have been executed as a basis for this procedure. In Figure 6 the data are plotted as dimensionless mean overtopping rates $Q = q/(g \cdot H^3)^{1/2}$ as a function of the relative freeboard $R = R_c/H$.

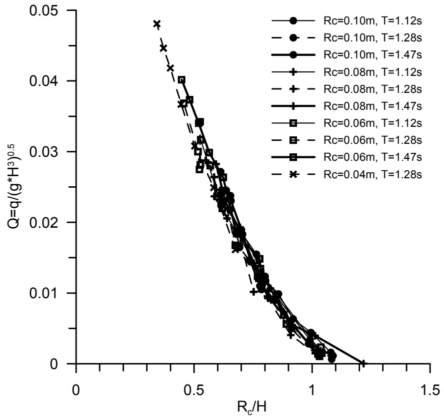

Figure 6. Dimensionless overtopping rates Q at a vertical wall (regular waves)

To fit a function to the data, the formula of KIKKAWA et al. [2], which is derived from the weir-formula, was used:

$$q = \frac{2}{15} \cdot m \cdot \sqrt{g \cdot H^3} \cdot \sqrt{2} \cdot k^{3/2} \cdot \left(1 - \frac{R_c}{k \cdot H}\right)^{5/2}$$

Applying non-linear regression to our data set resulted in m = 0.35 and k = 1.36.

Insofar the following data have not been directly measured in model tests with irregular waves. However, the sufficiency of the method has been proved within various test-series.

Fig. 7 shows an example of mean overtopping rates at a vertical wall for time-series with 30 and 1000 waves in comparison to design functions. For each relative freeboard investigated 64 tests were simulated.

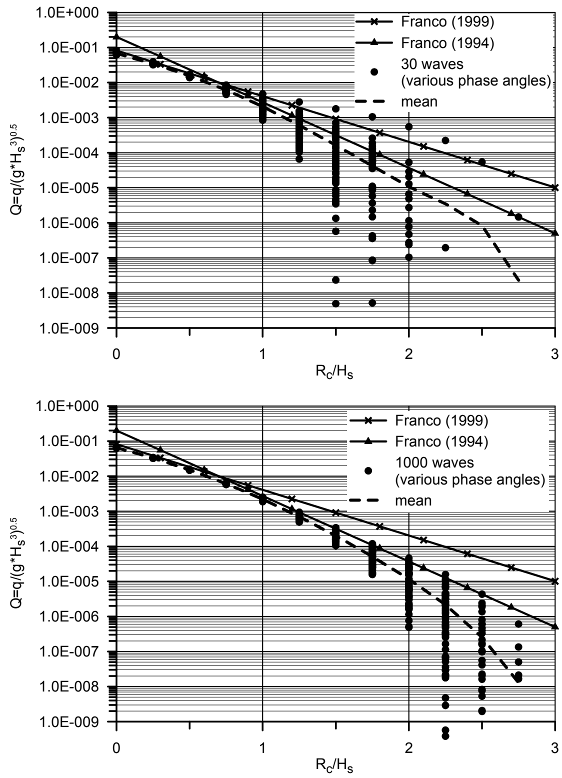

Figure 7. Variation of mean overtopping rates from various time-series top: 30 waves; bottom: 1000 waves

As to be expected, the scatter of the overtopping rates is larger for the time-series with 30 waves. However, even with time-series of 1000 waves there is a considerable scatter for R_c/H_s > about 1.5.

These results from a very large number of tests (usually not performed in standard investigations) highlight the problem of deriving design formulae from scattered data with only a few tests.

From a plot of the coefficients of variation (relative standard deviations) for various numbers of waves (Fig. 8) the increase of the relative scatter with increasing freeboard and the influence of the number of waves in a wave train can be seen.

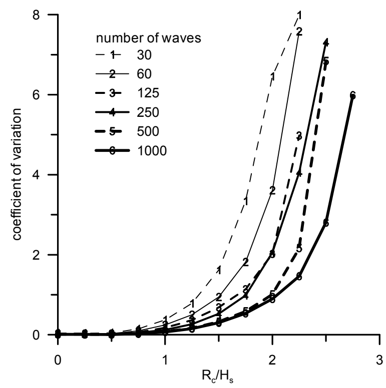

Figure 8. Coefficients of variation for various numbers of waves and relative freeboards

From these results it becomes clear that we need a certain number of tests (depending on the relative freeboard and the number of waves in the model wave train) to end up with reliable design formulas. For more details see Daemrich et al., 2010 [3].

II. Scatter due to the choice of the reference wave height at the toe of sloped sea dikes without considering energy flux (recommended in the more recent literature)

A. General

In the more recent literature it is recommended to use H_{m0} at the toe of sloped sea dikes as the reference wave height parameter for run-up and overtopping measurements and calculations with design formulas. However, it will be shown, that basic model tests or measurements may lead to different results for same wave heights, when investigations are performed in different water depths.

This can also be comprehended vividly, assuming shoaling (and refraction) to be similar off-shore and at the structure up to the point of wave breaking. The wave height varies due to shoaling in decreasing water depth. Run-up R_u and related overtopping q is dependent on the breaker wave height. Whereas the run-up is unique, the reference wave height varies with the reference water depth. Therewith the coefficient R/H used in design formulas is a function of the water depth. To get a unique coefficient, holding for any depth at the tow of a structure, only breaker wave height or deep water wave height are suitable. Because of the uncertainties in the measurement of breaking wave heights, the deep water wave height is preferred. Therefore, the use of the traditional deepwater related wave height H_{m0} / K_s (K_s = shoaling coefficient) is strongly recommended as reference wave height at sloped structures.

The influence of refraction is to be explained similarly. The wave direction in the area of the structure is changed by refraction from the toe of the structure to the breaking point. Different water depths at the toe of the structure relate to different changes of the direction up to the breaker point. As the direction in the breaker point is the unique parameter controlling the reduction of run-up, the wave direction at the toe is again not a unique parameter.

Due to a number of reasons the wave direction cannot be related to a deepwater wave direction. Therefore it is crucial to report the relative water depth of the investigations when recommending directional functions for reduction of run-up or overtopping.

B. Expected influence on the design formula from EurOtop Assessment Manual

The influence of the reference wave height parameter is demonstrated by varying the reference water depth and applying the currently recommended design formula from EurOtop. The range of results for an example of a sloped sea dike (1 : 6) is given in Fig. 9. The range of possible influence exceeds the difference between the formulas recommended for deterministic and probabilistic design.

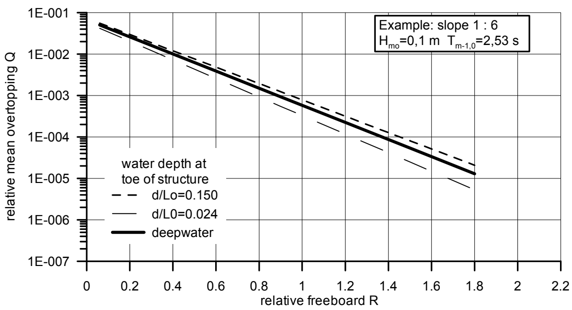

Figure 9. Influence of various model water depth on overtopping data at a sea dike

C. Overtopping measurements in regular waves (slope 1 : 6, perpendicular wave approach)

As a basis for the investigations of reasons of scatter in overtopping rates at sloped sea dikes, tests with regular (monochromatic) waves have been performed. The results were plotted as dimensionless mean overtopping rates Q as a function of the dimensionless freeboard R.

Because of the dependency of run-up R_u from the breaker parameter ξ_0 ($R_u \approx H \cdot \xi_0$), consequently ξ_0 is included in the dimensionless parameters. We have used the dimensionless parameters R and Q according the definition in EurOtop for sloped structures and breaking waves:

$$R = \frac{R_c}{H \cdot \xi_0} \qquad Q = \frac{q}{\sqrt{gH^3}} \frac{\sqrt{\tan\alpha}}{\xi_0}$$

$$\xi_0 = \frac{\tan\alpha}{\sqrt{H / L_0}} \qquad \alpha = \text{angle of slope}$$

To demonstrate the influence of the reference wave height, the measured data have been plotted for comparison with actual wave height H at the structure toe and with deepwater related wave height $H_0 = H / K_s$. From the plots in Fig. 10 it is clearly to be seen that the use of H_0 results in less scatter and closer functional coherence and is to be preferred for this type of structures.

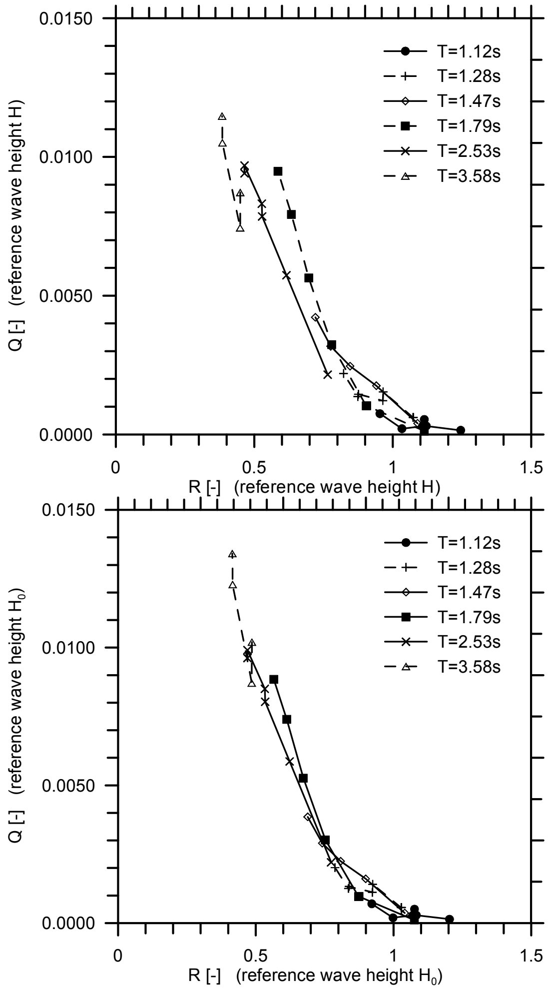

Figure 10a. Overtopping rates at dikes (slope 1 : 6, regular waves) Influence of the reference wave height on the scatter and functional coherence (linear vertical scale)
top: reference wave height H bottom: reference wave height H_0

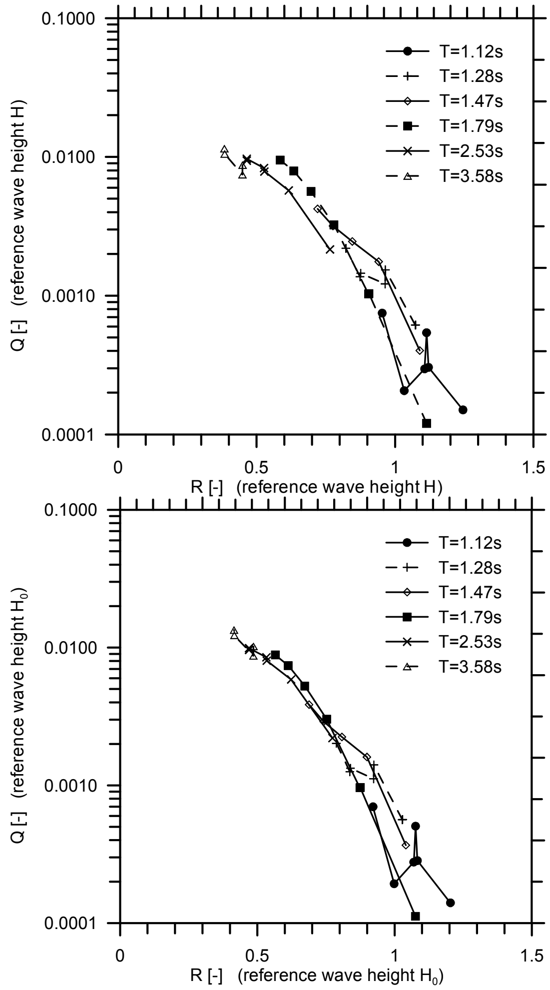

Figure 10b. Overtopping rates at dikes (slope 1 : 6, regular waves) Influence of the reference wave height on the scatter and functional coherence (logarithmic vertical scale)
top: reference wave height H bottom: reference wave height H_0

D. Run-up measurements in irregular waves (slope 1 : 6, oblique wave approach)

In general, the influence of the wave direction on the wave run-up height is expressed by a reduction factor γ_θ which is related to the perpendicular wave approach ($\theta = 0°$). The wave run-up height $R_{u2\%}$ for perpendicular wave approach is characterized by the following formula:

$$\frac{R_{u2\%}}{H_S} = 1.6\xi_{0p}$$

with $R_{u2\%}$ = wave run-up height exceeded by 2% of the incoming waves. The factor 1.6 is determined by hydraulic model tests.

The design formula for the wave run-up height $R_{u2\%}$ under consideration of oblique wave attack includes a reduction factor γ_θ:

$$\frac{R_{u2\%}}{H_S} = 1.6 \cdot \gamma_\theta \cdot \xi_{0p}$$

This reduction factor has been investigated by a number of researchers. Schüttrumpf, 2001 [4] has compiled functions of reduction factors (Fig. 11), showing a wide variety.

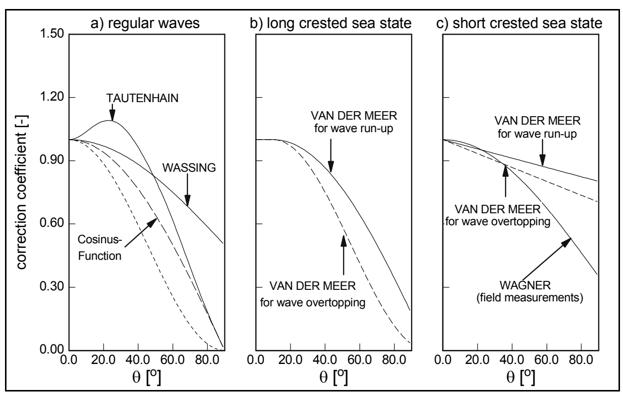

Figure 11. Influence of various model water depth on overtopping data at a sea dike (Schüttrumpf, 2001)

Because of such contradictorily results a research program on "Oblique wave run-up on sea dikes" (BMBF 03KIS015/016) was promoted and completed at the end of 2002. Reflections on the influence of the reference water depth with respect to shoaling and refraction go back to these investigations. The following results were published e.g. in Daemrich et al., 2004 [5].

Within this research program comparing experiments with short-crested and long-crested waves were performed in the wave basin of the Canadian Hydraulic Centre (CHC) of the National Research Centre (NRC) in Ottawa. A dike with a constant 1 : 6 slope was used in the experiments. Water depth was always 0.5 m, wave heights H_{m0} were 0.1 m, range of peak periods T_p from 1.27 s to 2.53 s.

The results of the investigations presented in Figure 12 are related to the measured wave parameters (H_{m0}, T_p, wave direction θ) at the toe of the dike. The coefficients of the best fit function for γ_θ of the type $y = a \cdot \cos x + b$ were determined by non-linear regression.

From these measurements with a dike of 1 : 6 constant slope, the function for γ_θ (with respect to the wave parameters at the toe of the dike) was determined from measured data for wave directions θ = 0°- 40° to:

$$\gamma_\theta = 0.67 \cdot \cos\theta + 0.33$$

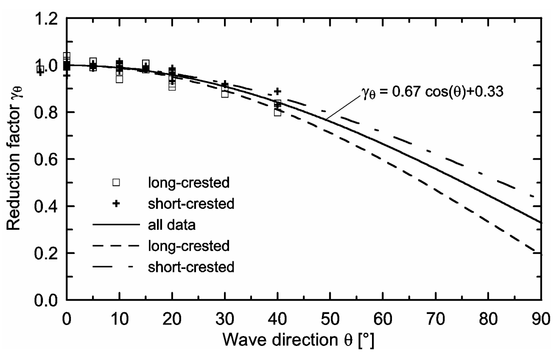

Figure 12: Reduction factor γ_θ determined from measurements in the wave basin of CHC, Canada (dike with constant slope 1 : 6, water depth at the toe 0.5 m, T_p = 1.27 s to 2.53 s)

With theoretical calculations of shoaling and refraction based on the linear wave theory the expected change of the significant wave heights and directions up to the breaker area were estimated for each data point. These data and the related best fit functions are shown in Fig. 13. Referring to the wave parameters in the breaking zone, the reduction coefficient γ_θ turned out to be approximate a pure cosine term (Figure 13).

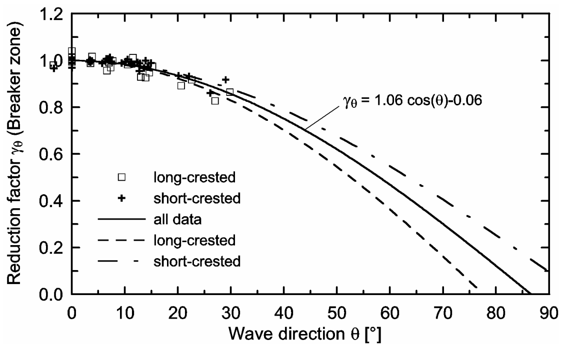

Figure 13. Reduction factor γ_θ considering refraction and shoaling along the dike up to the breaker zone (same data as Fig. 12)

Using a cosine function for γ_θ at the breaking point as an allegation, the reduction coefficient γ_θ related to the input parameters at the toe of the dike could be calculated theoretically as shown in Figure 14.

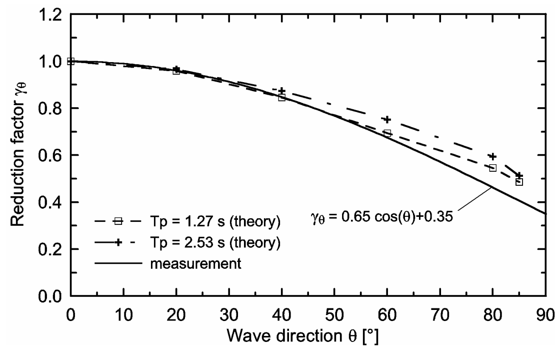

Figure 14. Theoretically expected directional function in comparison to the fit function of the measurements (dike slope 1 : 6, water depth at the toe 0.5 m)

This again is a strong indicator that functions of reduction factors in design formulas to consider oblique wave run-up (or wave overtopping) depend on the reference water depth.

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