Reprint: *„Mesh Generation for RMA2/FESWMS Based on B-Spline Surfaces“*. 3rd International Symposium on Environmental Hydraulics (ISEH), 05.- 08. December 2001, Tempe, Arizona, USA.

Mesh Generation for RMA2/FESWMS Based on B-Spline Surfaces

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1 INTRODUCTION

The numerical solution of environmental hydraulics require high quality surface representations of bathymetry, which has to be developed from different sets of scattered data points and to be fitted using additional information (e.g. edge information, structures, data quality). Regions of measured data points may overlap or gaps may appear, where significant surface information is not available. Some regions show an overabundance of data coming from different field campaigns using different measurement equipment. Until now, model geometry as the most important parameter set needed for numerical hydrodynamic modeling, is the basis to solve sediment and pollutant transport in natural aquatic ecosystems. Thus, b-spline approximation is developed to setup finite element meshes of high quality.

2 B-SPLINE SURFACES

De Boor developed the mathematical basics of b-spline curves and surfaces. These techniques of geometry modeling are widely applied in engineering and computer science [1][2]. A point $\mathbf{b}(u,v)$ on a b-spline surface in parameterized form is given in dependence of a global parameter set u and v:

$$\mathbf{b}(u,v) = \sum_{i=0}^{N} \sum_{j=0}^{M} \mathbf{d}_{ij} N_i^K(u) N_j^L(v) \quad \text{for} \quad u \in \left[u_K, u_{N+1}\right];\, v \in \left[v_L, v_{M+1}\right] \qquad (1)$$
$$\text{with} \quad N,\, M \in N \;\; ; \;\; K, L \in N \;\; ; \mathbf{d}_{ij} \in E^3$$

The shape of this segmented surface given in (1) is defined by control points $\mathbf{d}_{ij}$ and b-spline functions $N_i^K(u)$ and $N_j^L(v)$. The control points $\mathbf{d}_{ij}$ of the b-spline surface are called de Boor points and ordered in a regular grid with $N+1$ de Boor points in u parameter direction and $M+1$ de Boor points in v parameter direction. K and L denote the grade of b-spline functions in u and v parameter direction. These b-spline functions depend on knot vectors $\mathbf{u}$ in u parameter direction and $\mathbf{v}$ in v parameter direction. The knot vector $\mathbf{u}$ is defined as

$$\mathbf{u} = \left[u_0, u_{N+K+1}\right]^{\mathrm{T}} \quad \text{with} \quad u_i \le u_{i+1} \quad \text{for} \quad i = 0, \dots, N+K \qquad (2)$$

where the knots u_i represent the margins of the parameter segments. These knots influence the shape of the b-spline functions $N_i^K(u)$ and $N_j^L(v)$, which are defined by recursive formulas:

$$N_i^r(u) = \frac{u-u_i}{u_{i+r}-u_i} N_i^{r-1}(u) + \frac{u_{i+r+1}-u}{u_{i+r+1}-u_{i+1}} N_{i+1}^{r-1}(u) \quad \text{for} \quad r=1,...,N+K \ ; \quad i=0,...,N+K-r$$
$$N_i^0(u) = \begin{cases} 1.0 \ \text{for} \ u \in \left[u_i, u_{i+1}\right[\\ 0.0 \ \text{else} \end{cases} \quad \text{for} \quad i=0,...,N+K \tag{3}$$

The knot vector $\mathbf{v}$ and the b-spline functions $N_j^0(v)$ and $N_j^r(v)$ are given in analogy to formulas (2) - (3). In general, the margins of the b-spline surface are not equal to the margins of the de Boor grid. To guarantee this important property, the node vectors have to be modified:

$$\begin{aligned} u_0 &= ... = u_K \text{ and } u_N = ... = u_{N+K+1} \\ v_0 &= ... = v_L \text{ and } u_M = ... = u_{M+L+1} \end{aligned} \tag{4}$$

3 Bathymetry Approximation with B-Spline Surfaces

Approximation of bathymetry by measured points is described in [3]. An approximating b-spline surface is defined by regular grid of de Boor points and the knot vectors. These knot vectors are chosen equidistant regarding the restriction given in equation (4). At least $(N+1)*(M+1)$ measurement points $\mathbf{p}$ have to be given in order to avoid an under-determined set of equations. In practice, the number of measurement points is considerably higher.

At first the de Boor points are generated in regular and equidistant grid. At this state of the approximation process the $z-$coordinates have still the starting value 0. Measurement points $\mathbf{p}_m$ can be expressed as points of the approximating b-spline surface:

$$\mathbf{p}_m = \mathbf{b}(u_m, v_m) \quad \text{for} \quad 0 \le m \le NOMP \tag{5}$$

Equation (5) represent a set of three equations for the x, y and z coordinates. Using (5) to determine u_m and v_m values on the b-spline surface x and $y-$coordinates leads to a set of two equations, which is solved iteratively, since the coordinates of measurement points p_{xm} and p_{ym} and the coordinates of all de Boor points d_{xij} and d_{yij} are known:

$$\begin{aligned} p_{xm} &= b_x(u_m, v_m) = \sum_{i=0}^{N} \sum_{j=0}^{M} d_{xij} N_i^K(u_m) N_j^L(v_m) \\ p_{ym} &= b_y(u_m, v_m) = \sum_{i=0}^{N} \sum_{j=0}^{M} d_{yij} N_i^K(u_m) N_j^L(v_m) \end{aligned} \tag{6}$$

After u_m and v_m are determined, the only unknown values in (6) are the $z-$coordinates of de Boor points. Equation (7) is an over-determined set of equations with $(N+1)*(M+1)$ unknown $z-$coordinates d_{zij} and is solved by a Householder transformation.

$$p_{zm} = b_z(u_m, v_m) = \sum_{i=0}^{N}\sum_{j=0}^{M} d_{zij} N_i^K(u_m) N_j^L(v_m) \tag{7}$$

4 MESH GENERATION FROM APPROXIMATED B-SPLINE SURFACES

The generation of finite element meshes from approximated b-spline surfaces is explained for a section of a hydrodynamic model used for the Port of Bremen at the tidal part of the Weser River. Figure 1 shows the access channel of the main basin (Neustadt Harbor, right) and a part of the Weser River (Unterweser-km 8, left). The Weser River is described by measurement points gained from echo soundings (several campaigns in 1997, averaged values). For the access channel different records from dredging surveys had to be used. They were taken over a period of one year for the different sections. A sheet pile wall between access channel and Weser River to guide currents and sediment transport had to be incorporated as a single structure. Available data contains gaps and overlapping areas.

At first the de Boor point grid is defined (fig. 2). The $z-$coordinates in figures 1 to 4 are scaled with the factor of four in order to demonstrate clearly the characteristic of the bathymetry.

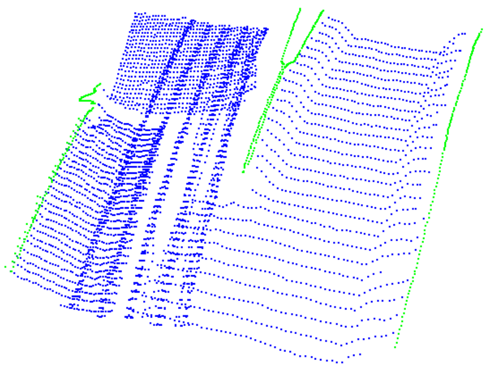

Figure 1: Measurement points

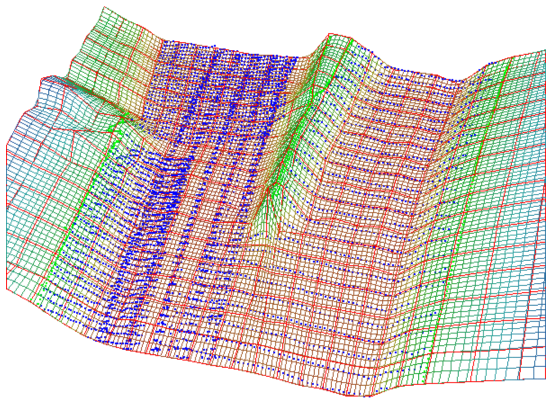

Figure 2: Measurement points, de Boor point grid and b-spline surface

After definition of the b-spline surface, a starting triangle mesh is generated with nodes on isolines defined by the user. The nodes are adapted to the border polygon represented in figure 3 as green points. Specified mesh refinement between $z = -10.0$ m and $z = -2.5$ m is performed now to get an optimized mesh for the tidal part (wet/dry problem in RMA2). In addition all elements outside the border polygon are deleted (figure 4).

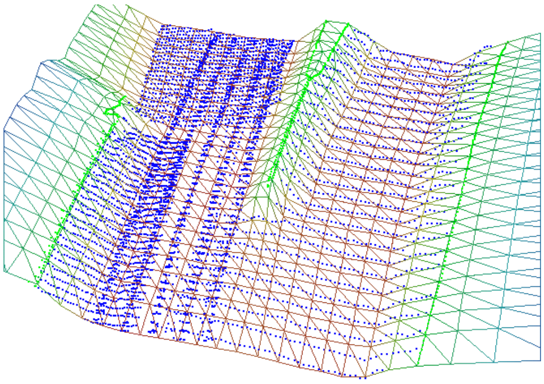

Figure 3: Starting triangle mesh

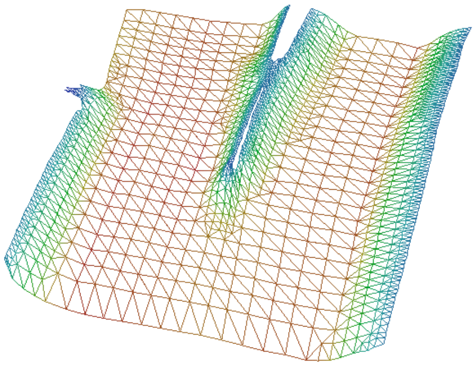

Figure 4: Refined mesh

Additional features allow the user in a dynamic process of mesh optimization to refine the mesh in distinct areas, re-define the b-spline parameter set or use other border polygons.

5 COMPARISON OF MESH SETUP, MESH QUALITY AND MODEL RESULTS

To compare the process of conventional geometry modeling with b-spline approximation and to get a first view to mesh quality, three different meshes for a hydrodynamic model of the Port of Bremen to simulate hydrodynamics and sediment transport are compared using the same data sets (echo soundings, geo-referenced section data and structures from ACAD files) for mesh setup. Roughness parameters were taken from a calibrated model (M1) and adopted/translated to calculated model geometries. The comparison of the different techniques is focused here on a Weser River section (Unterweser-km 7.5 to 8.5) of the model and the main basin (Neustadt Harbor). Data about mesh setup process and stability of resulting models is shown in table1.

Table 1: Mesh dimensions, needed model runs and stability after mesh generation

Mesh	Nodes/Elements	Correction of Data	Model Runs	Stability
M1	35662/16071	Intensive	15* / 10**	++
M2	35595/16024	Ignored	14* / no calibration	-
M3	50659/23506	not necessary	2* /no calibration	+

*Model runs for stability   ** necessary model runs to calibrate the model

Conventional techniques were used for the mesh M1 to eliminate overlapping areas, filter out z-coordinate errors and put in structures (e.g. sheet pile walls, quays). Embankments were put in “by hand” from digitized geo-referenced section data. Node topology was build in a first step by automated SMS mesh generation algorithm (triangulation) using field data. Node positions were optimized “by hand” (moving nodes and create quadrilateral elements from triangles) to get element topology parallel to isolines. In a final step original data was linear re-interpolated to the formerly defined mesh. Intensive calibration for a campaign of four weeks in 1990 and adoption of the mesh led to an optimized mesh, capable to simulate different discharge conditions and tidal situations[4, 5].

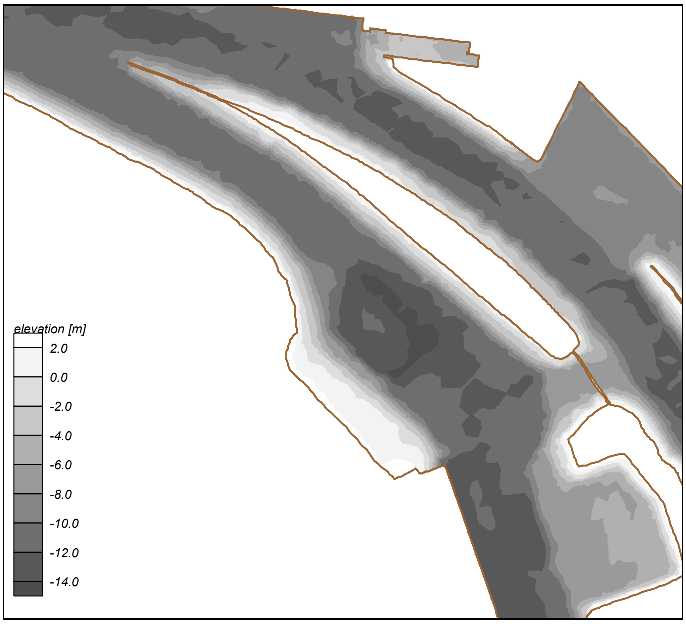

Figure 5: Bathymetrie of mesh M2

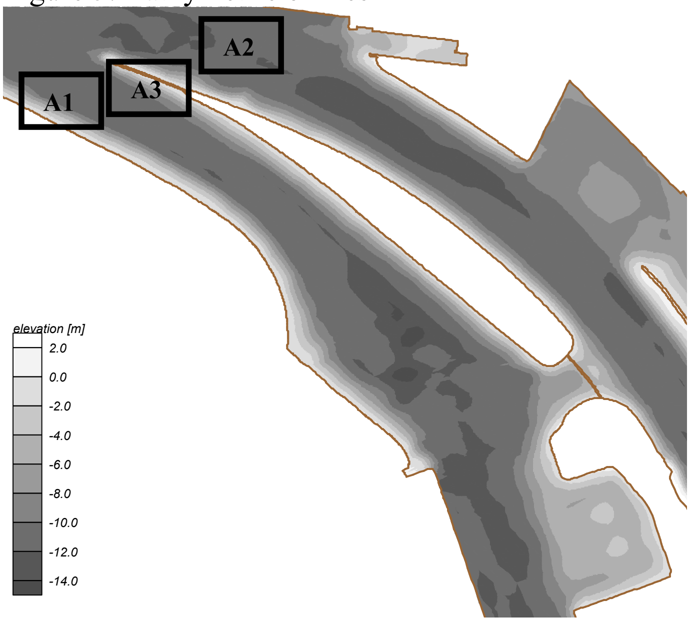

Figure 6: Bathymetrie of mesh M3

Modeling process for the second mesh M2 (Fig. 5) stopped after the import of original data and interpolation to nodes created by SMS. Only small adoptions were performed to get the model stable for the different hydrodynamic situations. M3 (Fig. 6) was setup by b-spline interpolation, using the process described above. Small adoptions and refinements were necessary to get the model stable, without having a look to field data used for calibration of M1 model. Comparison of M3 with the reference model M1 shows enhanced capabilities of b-spline approximation in areas with overlapping data and in the main river, where sufficient data exists. Problems occur near structures (e.g. sheet pile wall and quays) and embankments, where definition of the boundary polygon influences the interpolation. M2 gives a diffuse bathymetry in areas with overlapping echo soundings and embankments with few data input (Fig. 5). Table 2 summarizes differences in bathymetry calculation compared with selected areas in M1 (reference model).

Table 2: Differences in bathymetry calculation compared with areas in M1

Area (in Fig. 6)	M1/M2*	M1/M3*
Embankment - A1	0,95 (op)	1,09 (up)
Sheet pile wall - A2	1,11 (up)	1,42 (up)
River - A3	1,02 (up)	1,01 (up)

* averaged values

Characteristic discharge conditions [4, 5] were simulated to compare hydrodynamic results of RMA2. Fig. 7 and 8 show the influence of interpolation errors in M3 and indicate, that revision of b-spline approximation is also necessary near structures and embankments.

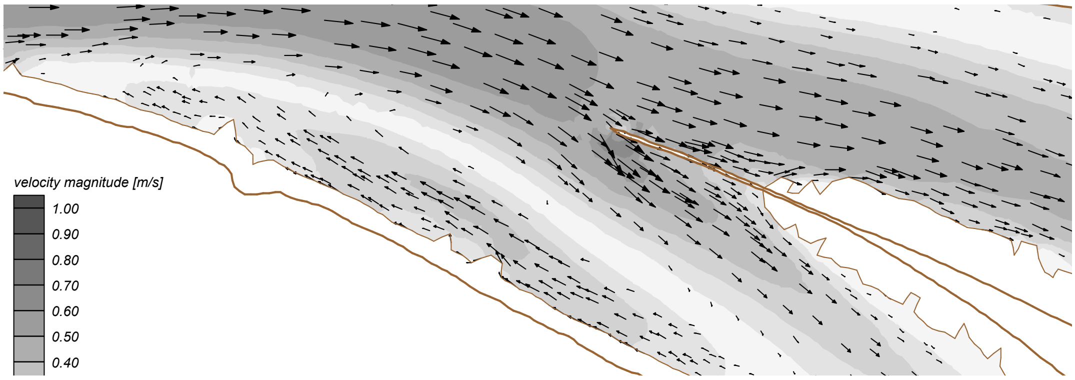

Figure 7: Model results for M1 (Q=150m^3/s at gauge Intschede, TS=14.5)

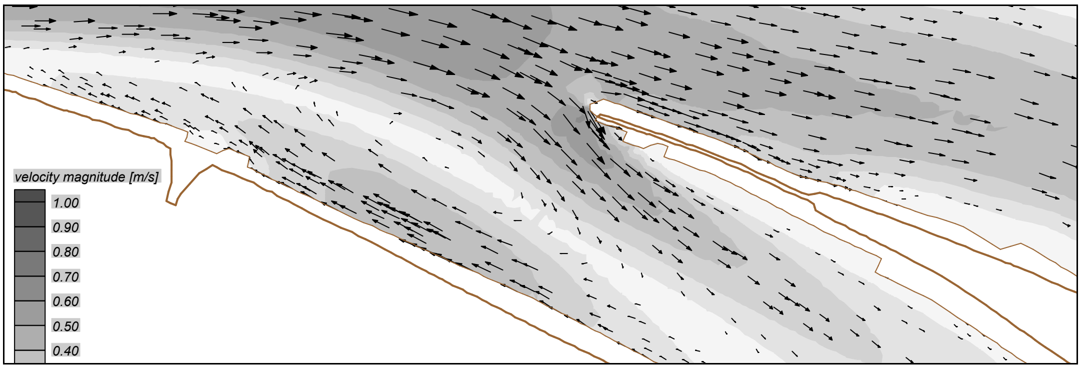

Figure 8: Model results for M3 (Q=150m^3/s at gauge Intschede, TS=14.5)

6 CONCLUSION

The present result of the software engineering process is an object oriented tool for b-spline approximation with comfortable GUI, dynamic memory allocation and an interface to RMA2 and FESWMS. Its development and following tests have shown applicability, limitations and necessary future developments. Advantages of the presented method are the flexibility to adopt the mesh generation process to specific needs and the short time needed to produce accurate results incorporating different data sources without revision of input data. Other element types, export facilities (e.g. PATRAN, IGES, STL and FE meshes for MIKE3), optimized memory handling and speed optimization will be topics for future development.

7 BIBLIOGRAPHY

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